Chapter Thirteen

Finite Series

The term 'order' is widely used in our day to day life. Such as, the concept of order is used to arrange the commodities in the shops, to arrange the incidents of drama and ceremony, to keep the commodities in attractive way in the godown. Again, to make many works easier and attractive, we use large to small, child to old, light to originated heavy etc. types of order. Mathematical series have been of all these concepts of order. In this chapter, the relation between sequence and series and contents related to them have been presented.

At the end of this chapter, the students will be able to –

- Describe the sequence and series and determine the difference between them
- Explain finite series
- Form formulae for determining the fixed term of the series and the sum of fixed numbers of terms and solve mathematical problems by applying the formulae
- Determine the sum of squares and cubes of natural numbers
- Solve mathematical problems by applying different formulae of series
- Construct formulae to find the fixed term of a geometrical progression and sum of fixed numbers of terms and solve mathematical problems by applying the formulae.

Sequence

Let us note the following relation :

Here, every natural number n is related to twice the number $2n$. That is, the set of positive even numbers $\{2, 4, 6, 8, ...\}$ is obtained by a method from the set of natural numbers $N = \{1, 2, 3, ...\}$. This arranged set of even number is a sequence. Hence, some quantities are arranged in a particular way such that the antecedent and subsequent terms becomes related. The set of arranged quantities is called a sequence.

The aforesaid relation is called a function and defined as $f(n) = 2n$. The general term of this sequence is $2n$. The terms of any sequence are infinite. The way of writing the sequence with the help of general term is $< 2n >, n = 1, 2, 3, ...$ or, $\{2n\}_{n=1}^{+\infty}$ or, $< 2n >$

The first quantity of the sequence is called the first term, the second quantity is called second term, the third quantity is called the third term etc. The first term of the sequence 1, 3, 5, 7... is 1, the second term is 2 etc.

Followings are the four examples of sequence :

$1, \quad 2, \quad 3, \ldots\ldots\ldots\ldots, n, \ldots\ldots\ldots\ldots$

$1, \quad 3, \quad 5, \ldots\ldots\ldots\ldots, (2n-1), \ldots\ldots\ldots$

$1, \quad 4, \quad 9, \ldots\ldots\ldots\ldots, n^2, \ldots\ldots\ldots\ldots$

$\frac{1}{2}, \quad \frac{2}{3}, \quad \frac{3}{4}, \ldots\ldots\ldots\ldots, \frac{n}{n+1}, \ldots\ldots\ldots\ldots$

Activity : 1. General terms of the six sequences are given below. Write down the sequences :

$(i)\ \frac{1}{n} \quad (ii)\ \frac{n-1}{n+1} \quad (iii)\ \frac{1}{2^n} \quad (iv)\ \frac{1}{2^{n-1}} \quad (v)\ (-1)^{n+1}\frac{n}{n+1} \quad (vi)\ (-1)^{n-1}\frac{n}{2n+1}$.

2. Each of you write a general term and then write the sequence.

Series

If the terms of a sequence are connected successively by + sign, a series is obtained. Such as, $1+3+5+7+\ldots\ldots$ is a series. The difference between two successive terms of the series is equal. Again, $2+4+8+16+\ldots\ldots$ is a series. The ratio of two successive terms is equal. Hence, the characterstic of any series depends upon the relation between its two successive terms. Among the series, two important series are arithmetic series and geometric series.

Arithmetic series

If the difference between any term and its antecedent term is always equal, the series is called arithmetic series.

Example : $1+3+5+7+9+11$ is a series. The first term of the series is 1, the second term is 3, the third term is 5 etc.

Here, second term − first term = $3-1=2$, third term − second term = $5-3=2$, fourth term − third term = $7-5=2$, fifth term − fourth term = $9-7=2$, sixth term − fifth term = $11-9=2$.

Hence the series is an arithmetic series. In this series, the difference between two terms is called common difference. The common difference of the mentioned series is 2. The numbers of terms of the series are fixed. That is why the series is finite series. It is to be noted that if the terms of the series are not fixed, the series is called infinite series, such as, $1+4+7+10+\ldots\ldots$ is an infinite series. In an arithmetic series, the first term and the common difference are generally denoted by a and d respectively. Then by definition, if the first term is a, the second term is $a+d$, the third term is $a+2d$, etc. Hence, the series will be $a+(a+d)+(a+2d)+\ldots$

Determination of common term of an arithmetic series

Let the first term of arithmetic series be a and the common difference be d, terms of the series are :

first term $= a \quad = a + (1-1)\,d$

second term $= a + d \quad = a + (2-1)\,d$

third term $= a + 2d \quad = a + (3-1)\,d$

forth term $= a + 3d \quad = a + (4-1)\,d$

....

....

$\therefore$ nth term $= a + (n-1)d$

This nth term is called common term of arithmetic series. If the first term of an arithmetic series in a and common difference is d, all the terms of the series are determined successively by putting $n = 1, 2, 3, 4, \ldots \ldots$ in the nth term.

Let the first term of an arithmetic series be 3 and the common difference be 2. Then second term of the series $= 3 + 2 = 5$, third term $= 3 + 2 \times 2 = 7$, forth term $= 3 + 3 \times 2 = 9$ etc.

Therefore, nth term of the series $= 3 + (n-1) \times 2 = 2n + 1$.

> **Activity :** If the first term of an arithmetic series is 5 and common difference is 7, find the first six terms, 22nd term, r th term and $(2p+1)$th term.

Example 1. Of the series, $5 + 8 + 11 + 14 + \cdots \cdots$ which term is 383 ?

Solution : The first term of the series $a = 5$, common difference $d = 8 - 5 = 11 - 8 = 3$

$\therefore$ It is an arithmetic series.

Let, nth term of the series $= 383$

We know that, nth term $= a + (n-1)d$.

$\therefore \quad a + (n-1)d = 383$

or, $5 + (n-1)3 = 383$

or, $5 + 3n - 3 = 383$

or, $3n = 383 - 5 + 3$

or, $3n = 381$

or, $n = \dfrac{381}{3}$

$\therefore \quad n = 127$

$\therefore$ 127^{th} term of the given series $= 383$.

Sum of *n* terms of an Arithmetic series

Let the first term of any arithmetic series be a, last term be p, common difference be d, number of terms be n and sum of n numbers of terms be S_n.

Writing from the first term and conversely from the last term of the series we get,

$$S_n = a + (a+d) + (a+2d) + \cdots \cdots + (p-2d) + (p-d) + p \qquad (i)$$

and $$S_n = p + (p-d) + (p-2d) + \cdots \cdots + (a+2d) + (a+d) + a \qquad (ii)$$

Adding, $2S_n = (a+p) + (a+p) + (a+p) + \ldots \ldots + (a+p) + (a+p) + (a+p)$

or, $2\,S_n = n(a+p)$ [$\because$ number of terms of the series is n]

$$\therefore \quad S_n = \frac{n}{2}(a+p) \qquad (iii)$$

Again, nth term $= p = a + (n-1)d$. Putting this value of p in (iii) we get,

$$S_n = \frac{n}{2}[a + \{a + (n-1)d\}]$$

i.e., $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$ $\qquad (iv)$

If the first term of arithmetic series a, last term p and number of terms n are known, the sum of the series can be determined by the formula (iii). But if first term the a, common difference d, number of terms n are known, the sum of the series are determined by the formula (iv).

Determination of the sum of first *n* terms of natural numbers

Let S_n be the sum of n numbers of natural numbers i.e.

$$S_n = 1 + 2 + 3 + \cdots\cdots\cdots + (n-1) + n \qquad (i)$$

Writing from the first term and conversely from the last term of the series we get,

$$S_n = 1 + 2 + 3 + \cdots\cdots\cdots + (n-2) + (n-1) + n \qquad (i)$$

and $\quad S_n = n + (n-1) + (n-2) + \cdots\cdots + 3 + 2 + 1 \qquad (ii)$

Adding, $2S_n = (n+1) + (n+1) + (n+1) + \cdots\cdots\cdots + (n+1)$ [n number of terms]

or, $\quad 2S_n = n(n+1)$

$$\therefore \quad S_n = \frac{n(n+1)}{2} \qquad (iii)$$

Example 2. Find the sum total of first 50 terms of natural numbers.

Solution : Using formula (iii) we get,

$$S_{50} = \frac{50\,(50+1)}{2} = 25 \times 51 = 1275$$

$\therefore$ The sum total of first 50 natural numbers is 1275

Example 3. $1 + 2 + 3 + 4 + \cdots\cdots\cdots + 9 =$ what ?

Solution : The first term of the series $a = 1$, common difference $d = 2 - 1 = 1$ and the last term $p = 9$

$\therefore$ It is an arithmetic series.

Let the nth term of the series $= 9$

We know, nth term of an arithmetic progression $= a + (n-1)d$

$\therefore \quad a + (n-1)d = 9$

or, $\quad 1 + (n-1)1 = 9$

or, $\quad 1 + n - 1 = 9$

Alternative method :

Since

$$S_n = \frac{n}{2}(a + p,)$$

$$\therefore \; S_9 = \frac{9}{2}(1 + 9)$$

$\therefore \quad n = 9$

$= \dfrac{9 \times 10}{2} = 45$

From (iv) formula, the sum of first n terms of an arithmetic series

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Hence, the sum 9 terms of the series $S_9 = \dfrac{9}{2}\{2 \times 1 + (9-1) \times 1\} = \dfrac{9}{2}(2+8)$

$$= \frac{9 \times 10}{2} = 9 \times 5 = 45$$

Example 4. What is the sum of 30 terms of the series $7 + 12 + 17 + \cdots\cdots\cdots$

Solution : First term of the series $a = 7$, common difference $d = 12 - 7 = 5$

$\therefore$ It is an arithmetic series. Here, number of terms $n = 30$.

We know that the sum of n terms of an arithmetic series

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

So, the sum of 30 terms $S_{30} = \dfrac{30}{2}\{2.7 + (30-1)5\} = 15(14 + 29 \times 5)$

$$= 15(14 + 145) = 15 \times 159$$

$$= 2385$$

Example 5. A deposits Tk. 1200 from his salary in the first month and in every month of subsequent months, he deposits Tk. 100 more than the previous months.

(i) How much does he deposit in nth month ?

(ii) Express the aforesaid problem in series upto n terms.

(iii) How much does he deposit in first n months ?

(iv) How much does he deposit in a year ?

Solution : (i) In the first month, he deposits Tk. 1200

In the second month, he deposits Tk. $(1200 + 100)$ = Tk. 1300

In third month, he deposits Tk. $(1300 + 100)$ = Tk. 1400

In forth month, he deposits Tk. $(1400 + 100)$ = Tk. 1500

Hence, it is an arithmetic series whose first term is $a = 1200$, common difference $d = 1300 - 1200 = 100$.

nth term of the series $= a + (n-1)d$

$$= 1200 + (n-1)100 = 1200 + 100n - 100$$

$$= 100n + 1100$$

Therefore, he deposits Tk. $(100n + 1100)$ in nth month.

(ii) The series, in this case upto n numbers of terms will be

$120+130+140+\cdots\cdots\cdots+(110+10n)$

(iii) In n numbers of months, he deposits Tk. $\frac{n}{2}\{2a+(n-1)d\}$

$=$ Tk. $\frac{n}{2}\{2\times 120+(n-1)10\}$

$=$ Tk. $\frac{n}{2}(240+10n-10)=$ Tk. $\frac{n}{2}\times 2(115+5n)$

$=$ Tk. $n(5n+115)$

(iv) We know that 1 year $=12$ months. Here $n=12$.

Therefore, A deposits in 1 year $=$ Tk. $12(5\times 12+115)$

$=$ Tk. $12(60+115)$

$=$ Tk. 12×175

$=$ Tk. 2100.

Exercise 13.1

1. Find the common difference and the 12th terms of the series $2-5-12-19-\cdots$
2. Which term of the series $8+11+14+17+\cdots\cdots\cdots$ is 392 ?
3. Which term of the series $4+7+10+13+\cdots\cdots\cdots$ is 301 ?
4. If the p th term of an arithmetic series is p^2 and q th term is q^2, what is $(p+q)$ th term of the series ?
5. If the m th tem of an arithmetic series is n and n th term is m, what is $(m+n)$th term of the series ?
6. What is the number of n terms of the series $1+3+5+7+\cdots\cdots\cdots$?
7. What is the sum of first 9 terms of the series $8+16+24+\cdots\cdots\cdots n$?
8. $5+11+17+23+\cdots\cdots\cdots+95=$ What ?
9. $29+25+21+\cdots\cdots\cdots-23=$ What ?
10. The 12th term of an arithmetic series is 77. What is the sum of the first 23 terms?
11. If the 16th term of an arithmetic series is -20, what will be the sum of first 31 terms ?
12. The total sum of first n terms of the series $9+7+5+\cdots\cdots\cdots$ is -144. Find the value of n.
13. If the sum of first n terms of the series $2+4+6+8+\cdots\cdots\cdots$ is 2550, find the value of n.
14. If the sum of first n terms of the series is $n(n+1)$, find the series.
15. If the sum of first n terms of the series is $n(n+1)$, what is the sum of first 10 terms ?
16. If the sum of 12 terms of an arithmetic series is 144 and first 20 terms is 560, find the sum of first 6 terms.

17. The sum of the first m terms of an arithmetic series is n and the first n terms is m. Find the sum of first ($m+n$) terms.
18. If the pth, qth and rth terms of an arithmetic series are a, b, c, respectively, show that $a(q-r)+b(r-p)+c(p-q)=0$.
19. Show that, $1+3+5+7+\cdots\cdots\cdots+125 = 169+171+173+\cdots\cdots\cdots+209$.
20. A man agrees to refund the loan of Tk. 20 in some parts. Each part is Tk. 2 more than the previous part. If the first part is Tk. 1, in how many parts will the man be able to refund that amount ?

Determinaton of the sum of Squares of the first *n* numbers of Natural Numbers

Let S_n be the number of squares of the first n numbers of natural numbers

i.e., $S_n = 1^2+2^2+3^2+\cdots\cdots\cdots+n^2$

We know,

$$r^3-3r^2+3r-1=(r-1)^3$$

or, $r^3-(r-1)^3 = 3r^2-3r+1$

In the above identity, putting, $r=1, 2, 3, \ldots\ldots, n$ we get,

$$1^3-0^3 = 3.1^2-3.1+1$$
$$2^3-1^3 = 3.2^2-3.2+1$$
$$3^3-2^3 = 3.3^2-3.3+1$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots$$
$$n^3-(n-1)^3 = 3.n^2-3.n+1$$

Adding, we get,

$$n^3-0^3 = 3(1^2+2^2+3^2+\cdots\cdots\cdots+n^2)-3(1+2+3+\cdots\cdots\cdots+n)+(1+1+1+\cdots\cdots\cdots+1)$$

or, $n^3 = 3S_n - 3.\dfrac{n(n+1)}{2}+n \quad \left[\because\ 1+2+3+\cdots\cdots\cdots+n=\dfrac{n(n+1)}{2}\right]$

or, $3S_n = n^3+\dfrac{3n(n+1)}{2}-n$

$$=\frac{2n^3+3n^2+3n-2n}{2}=\frac{2n^3+3n^2+n}{2}=\frac{n(2n^2+3n+1)}{2}$$

$$=\frac{n(2n^2+2n+n+1)}{2}=\frac{n\{2n(n+1)+1(n+1)\}}{2}$$

or, $3S_n = \dfrac{n(n+1)(2n+1)}{2}$

$\therefore \quad S_n = \dfrac{n(n+1)(2n+1)}{6}$

The sum of cubes of the first *n* numbers of Natural Numbers

Let S_n be the sum of cubes of the first n numbers of natural numbers.

That is, $S_n = 1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3$

We know that, $(r+1)^2 - (r-1)^2 = (r^2 + 2r + 1) - (r^2 - 2r + 1) = 4r.$

or, $(r+1)^2 r^2 - r^2 (r-1)^2 = 4r.r^2 = 4r^3$ [Multiplying both the sides by r^2]

In the above identity, putting $r = 1, 2, 3, \ldots \ldots, n$

We get,

$2^2.1^2 - 1^2.0^2 = 4.1^3$
$3^2.2^2 - 2^2.1^2 = 4.2^3$
$4^2.3^2 - 3^2.3^2 = 4.3^3$
$\cdots \quad \cdots \quad \cdots \quad \cdots$
$\cdots \quad \cdots \quad \cdots \quad \cdots$
$(n+1)^2 n^2 - n^2 (n-1)^2 = 4n^3$

Adding, we get, $(n+1)^2.n^2 - 1^2.0^2 = 4(1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3)$

or, $(n+1)^2.n^2 = 4S_n$

or, $S_n = \dfrac{n^2(n+1)^2}{4}$

$\therefore \quad S_n = \left\{\dfrac{n(n+1)}{2}\right\}^2$

Necessary formulae :

$$1 + 2 + 3 + \cdots\cdots\cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots\cdots\cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

N.B: $1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3 = (1 + 2 + 3 + \cdots\cdots\cdots + n)^2$.

Activity : 1. Find the sum of natural even numbers of the first n numbers.
2. Find the sum of squares of natural odd numbers of the first n numbers.

Geometric series

If the ratio of any term and its antecedent term of any series is always equal i.e., if any term divided by its antecedent term, the quotient is always equal, the series is called a geometric series and the quotient is called common ratio. Such as, of the series $2+4+8+16+32$, the first term is 2, the second term is 4, the third term is 8, the fourth term is 16 and the fifth term is 32. Here, the ratio of the second term to the first term $=\frac{4}{2}=2$, the ratio of the third term to the second term $=\frac{8}{4}=2$, the ratio of the fourth term to the third term $=\frac{16}{8}=2$, the ratio of the fifth term to the fourth term $=\frac{32}{16}=2$.

In this series, the ratio of any term to its antecedent term is always equal. The common ratio of the mentioned series is 2. The numbers of terms of the series are fixed. That is why the series is finite geometric series. The geometric series is widely used in different areas of physical and biological science, in organizations like Bank and Life Insurance etc, and in different branches of technology. If the numbers of terms are not fixed in a geometric series, it is called an infinite geometric series.

The first term of a geometric series is generally expressed by a and common ratios by r. So by definition, if the first term is a, the second term is ar, the third term is ar^2, etc. Hence the series will be $a+ar+ar^2+ar^3+\cdots\cdots\cdots$

Activity : Write down the geometric series in the following cases :

(i) The first term 4, common ratio 10 (ii) The first term 9, common ratio $\frac{1}{3}$

(iii) The first term 7, common ratio $\frac{1}{10}$ (iv) The first term 3, common ratio 1

(v) The first term 1, common ratio $-\frac{1}{2}$ (vi) The first term 3, common ratio -1

General term of a Geometric series

Let the first term of a geometric series be a, and common ratio be r. Then, of the series,

first term $=a=ar^{1-1}$, second term $=ar=ar^{2-1}$

third term $=ar^2=ar^{3-1}$, fourth term $=ar^3=ar^{4-1}$

... ...

... ...

nth term $=ar^{n-1}$

This nth term is called the general term of the geometric series. If the first term of a geometric series a and the common ratio r are known, any term of the series can be determined by putting $r=1, 2, 3, \ldots\ldots$ etc. successively in the nth term.

Example 6. What is the 10th term of the series $2+4+8+16+\cdots\cdots$?

Solution : The first term of the series $a = 2$, common ratio $r = \frac{4}{2} = 2$.

$\therefore$ The given series is a geometric series.

We know that the nth term of geometric series = ar^{n-1}

$\therefore$ 10 th term of the series $= 2 \times 2^{10-1}$

$= 2 \times 2^9 = 1024$

Example 7. What is the general term of the series $128 + 64 + 32 + \cdots\cdots\cdots$?

Solution : The first term of the series $a = 128$, common ratio $r = \frac{64}{128} = \frac{1}{2}$.

$\therefore$ It is a geometric series.

We know that the general term of the series = ar^{n-1}

Hence, the general term of the series $= 128 \times \left(\frac{1}{2}\right)^{n-1} = \frac{2^7}{2^{n-1}} = \frac{1}{2^{n-1-7}} = \frac{1}{2^{n-8}}$.

Example 8. The first and the second terms of a geometric series are 27 and 9. Find the 5th and the 10th terms of the series.

Solution : The first term of the given series $a = 27$, the second term is 9.

Then the common ratio $r = \frac{9}{27} = \frac{1}{3}$.

$\therefore$ The 5th term = $ar^{5-1} = 27 \times \left(\frac{1}{3}\right)^4 = \frac{27 \times 1}{27 \times 3} = \frac{1}{3}$

and the 10th term = $ar^{10-1} = 27 \times \left(\frac{1}{3}\right)^9 = \frac{3^3}{3^3 \times 3^6} = \frac{1}{3^6} = \frac{1}{729}$.

Determination of the sum of a Geometric series

Let the first term of the geometric series be a, common ratio r and number of terms n. If S_n is the sum of n terms,

$S_n = a + ar + ar^2 + \cdots\cdots\cdots + ar^{n-2} + ar^{n-1}$ $\quad (i)$

and $r.S_n = \quad ar + ar^2 + ar^3 + \cdots\cdots\cdots + ar^{n-1} + ar^n$ [multiplying (i) by r] $\quad$ (ii)

Subtracting, $S_n - rS_n = a - ar^n$

or, $S_n(1-r) = a(1-r^n)$

$\therefore \quad S_n = \frac{a(1-r^n)}{1-r}$, when $r < 1$

Again, subtracting (ii) form (i) we get,

$rS_n - S_n = ar^n - a$ or, $S_n(r-1) = a(r^n - 1)$

i.e., $S_n = \frac{a(r^n-1)}{(r-1)}$, when $r > 1$.

Observe : If common ratio is $r = 1$, each term $= a$

Hence, in this case $S_n = a + a + a + \cdots \cdots$ upto n.
$= an$.

Activity : A employed a man from the first April for taking his son to school and taking back home for a month. His wages were fixed to be one paisa in first day, twice of the first day in second day i.e. two paisa, twice of the second day in the third day i.e. four paisa. If the wages were paid in this way, how much would he get after one month including holidays of the week ?

Example 9. What is the sum of the series $12 + 24 + 48 + \cdots \cdots \cdots + 768$?

Solution : The first term of the series is $a = 12$, common ratio $r = \frac{24}{12} = 2 > 1$.

$\therefore$ the series is a geometric series.

Let the nth term of the series $= 768$

We know, nth term $= ar^{n-1}$

$\therefore \quad ar^{n-1} = 768$

or, $\quad 12 \times 2^{n-1} = 768$

or, $\quad 2^{n-1} = \frac{768}{12} = 64$

or, $\quad 2^{n-1} = 2^6$

or, $\quad n - 1 = 6$

$\therefore \quad n = 7$.

Therefore, the sum of the series $= \frac{a(r^n - 1)}{(r-1)}$, when $r > 1$

$$= \frac{12(2^7 - 1)}{2-1} = 12 \times (128 - 1) = 12 \times 127 = 1524.$$

Example 10. Find the sum of first eight terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \cdots \cdots$

Solution : The 1st term of the series is $a = 1$, common ratio $r = \frac{\frac{1}{2}}{1} = \frac{1}{2} < 1$

$\therefore$ It is a geomeric series.

Here the number of terms $n = 8$.

We know, sum of n terms of a geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}, \text{ when } r < 1.$$

Hence, sum of eight terms of the series is $S_8 = \frac{1 \times \left\{1 - \left(\frac{1}{2}\right)^8\right\}}{1 - \frac{1}{2}} = \frac{1 - \frac{1}{256}}{\frac{1}{2}}$

$$= 2\left(\frac{256 - 1}{256}\right) = \frac{255}{128} = 1\frac{127}{128}$$

Exercise 13.2

1. If a, b, c, d are cosecutive four terms of a arithmetic series, which one is correct ?

   (a) $b = \frac{c+d}{2}$ (b) $a = \frac{b+c}{2}$ (c) $c = \frac{b+d}{2}$ (d) $d = \frac{a+c}{2}$

2. (i) If $a + (a+d) + a + 2d) + ...$ the sum of first n is terms of the series is $\frac{n}{2}\{2a + (n-1)d\}$

   (ii) $1 + 2 + 3 + ... + n = \frac{n(n+1)(2n+1)}{6}$

   (iii) $1 + 3 + 5 + ... + (2n-1) = n^2$

   Which one of the followings is correct according to the above statements.

   (a) i and ii (b) i and iii (c) ii and iiii (d) i, ii and iii

   Answer the questions 3 and 4 on the basis of following series :

   log 2 + log 4 + log 8 + ...

3. Which one is the common difference of the series ?

   (a) 2 (b) 4 (c) log 2 (d) 2 log 2

4. Which one is the 7th term of the series

   (a) log 32 (b) log 64 (c) log 128 (d) log 256

5. Determine the 8th term of the series 64 + 32 + 16 + 8 + ...

6. Determine the sum of first fourteen terms of the series 3 + 9 + 27 + ...

7. Which of the term is $\frac{1}{2}$ of the series 128 + 64 + 32 + ...

8. If the 5th terms of a geometric series $\frac{2\sqrt{3}}{9}$ and the 10th term are $\frac{8\sqrt{2}}{8}$, find the 3rd term of the series.

9. Which of the term is $8\sqrt{2}$ of the sequence $\frac{1}{\sqrt{2}}, -1, \sqrt{2}, \cdots \cdots \cdots$?

10. If $5 + x + y + 135$ is geometric series, find the value of x and y.

11. If $3 + x + y + z + 243$ is geometric series, find the value of x, y and z.

12. What is the sum of first seven terms of the series $2 - 4 + 8 - 16 + \cdots \cdots \cdots$?

13. Find the sum of $(2n+1)$ terms of the series $1 - 1 + 1 - 1 + \cdots \cdots \cdots$

14. What is the sum of first ten terms of the series $\log 2 + \log 4 + \log 8 + \cdots \cdots \cdots$?

15. Find the sum of first twelve terms of the series

    $\log 2 + \log 16 + \log 128 + \cdots \cdots$?

16. If the sum of n terms of the series $2 + 4 + 8 + 16 + \cdots \cdots \cdots$ is 254, what is the value of n ?

17. What is sum of $(2n+2)$ terms of the series $2-2+2-2+\cdots\cdots\cdots$?
18. If the sum of cubes of n natural numbers is 441, find the value of n and find the sum of those terms.
19. If the sum of cubes of the first n natural numbers is 225, find the value of n and find the sum of square of those terms ?
20. Show that $1^3+2^3+3^3+\ldots\ldots\ldots\ldots+n^3=(1+2+3+\ldots\ldots+n)^2$.
21. If $\dfrac{1^3+2^3+3^3+\cdots\cdots\cdots+n^3}{1+2+3+\cdots\cdots\cdots+n}=210$, what is the value of n ?
22. A iron-bar with length one metre is divided into ten pieces such that the lengths of the pieces form a geometric progression. If the largest pieces is ten times of the smallest one, find the length in approximate millimetre of the smallest piece.
23. The first term of a geometric series is a, common ratio is r, the fourth term of the series is -2 and the 9th term is $8\sqrt{2}$.

    (a) Express the above information by two equations.

    (b) Find the 12th term of the series.

    (c) Find the series and then determine the sum of the first seven terms of the series.
24. The nth term of The a series is $2n-4$.

    (a) Find the series.

    (b) Find the 10th term of the series and determine the sum of first 20 terms.

    (c) Considering the first term of the obtained series as 1st term and the common difference as common ratio, construct a new series and find the sum of first 8 terms of the series by applying the formula.